# MEAN FIELD GAMES AND SYSTEMIC RISK

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**Abstract.**
We propose a simple model of inter-bank borrowing and lending where the evolution of the log-monetary reserves of $N$ banks is described by a system of diffusion processes coupled through their drifts in such a way that stability of the system depends on the rate of inter-bank borrowing and lending. Systemic risk is characterized by a large number of banks reaching a default threshold by a given time horizon. Our model incorporates a game feature where each bank controls its rate of borrowing/lending to a central bank. The optimization reflects the desire of each bank to borrow from the central bank when its monetary reserve falls below a critical level or lend if it rises above this critical level which is chosen here as the average monetary reserve. Borrowing from or lending to the central bank is also subject to a quadratic cost at a rate which can be fixed by the regulator. We solve explicitly for Nash equilibria with finitely many players, and we show that in this model the central bank acts as a clearing house, adding liquidity to the system without affecting its systemic risk. We also study the corresponding Mean Field Game in the limit of large number of banks in the presence of a common noise.

**Key words.** Systemic risk, interbank borrowing and lending, stochastic games, Nash equilibrium, Mean Field Game.

**Subject classifications.** 60H30, 91A15, 91G20, 93E20

*Dedicated to George Papanicolaou in honor of his 70th birthday*

## 1. Introduction

Systemic risk is becoming a central research topic. We refer to the Handbook [8] for recent developments on systemic risk from many points of view (Statistics, Finance, Mathematical Finance, Behavioral Finance, Networks, Counterparty Risk, High Frequency Trading, ...). Here, we propose a simple model of inter-bank borrowing and lending where the evolution of the log-monetary reserves of $N$ banks is described by a system of diffusion processes coupled through their drifts in such a way that stability of the system depends on the rate of inter-bank borrowing and lending. Systemic risk is characterized by a large number of banks reaching a default threshold by a given time horizon. This type of interaction and the relation *stability–systemic risk* has been recently studied in [9], [10], [11], and [12]. Here, we introduce a game feature where each bank controls its rate of borrowing/lending to a central bank. The control of each individual bank reflects the desire to borrow from the central bank when its monetary reserve falls below a critical level or lend if it rises above this critical level which is chosen here as the average monetary reserve. Borrowing from or lending to the central bank is also subject to a quadratic cost at a rate which can be fixed by the regulator. As written, our model is an example of Linear-Quadratic Mean Field Game with finitely many players which can be solved explicitly. We first solve for open-loop equilibria using the Pontryagin stochastic maximum principle. We also solve for closed-loop equilibria using the probabilistic approach based on the Pontryagin stochastic maximum principle leading to the solution of Forward-Backward Stochastic Differential Equations, and the dynamic programming principle leading to

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