We consider default by firms that are part of a single clearing mechanism. The obligations of all firms within the system are determined simultaneously in a fashion consistent with the priority of debt claims and the limited liability of equity. We first show, via a fixed-point argument, that there always exists a “clearing payment vector” that clears the obligations of the members of the clearing system; under mild regularity conditions, this clearing vector is unique. Next, we develop an algorithm that both clears the financial system in a computationally efficient fashion and provides information on the systemic risk faced by the individual system firms. Finally, we produce qualitative comparative statics for financial systems. These comparative statics imply that, in contrast to single-firm results, even unsystematic, nondissipative shocks to the system will lower the total value of the system and may lower the value of the equity of some of the individual system firms.

(Credit Risk; Default; Clearing Systems)

1. Introduction
One of the most pervasive aspects of the contemporary financial environment is the rich network of interconnections among firms. Although financial liabilities owed by one firm to another are usually modeled as unidirectional obligations dependent only on the financial health of the issuing firm, in reality, the liability structure of corporate obligations is invariably much more intricate. The value of most firms is dependent on the payoffs they receive from their claims on other firms. The value of these claims depends, in turn, on the financial health of yet other firms in the system. Moreover, linkages between firms can be cyclical. A default by Firm A on its obligations to Firm B may lead B to default on its obligations to C. A default by C may, in turn, have a feedback effect on A. This example illustrates a general feature of financial system architectures, which we term cyclical interdependence. In this paper, we consider the problem of finding a clearing mechanism in cases in which this sort of cyclical interdependence is present.

All markets have some kind of clearing mechanism. Perhaps clearing mechanisms of interbank payments and for listed exchanges have received the most attention. In the United States, for example, CHIPS and Fedwire are the main banking clearing mechanisms; in Germany, the Abrechnung and the EAF (Elektronische Abrechnung mit Filetransfer) performs this function. Regarding clearing mechanisms, one of the attractions of trading on a listed options exchange, the CBOE, for example, is that the Options Clearing Corporation is the counterparty to every trade. Hence, credit considerations do not prohibit lower-credit traders from participating in these markets. These payment systems are forced to confront large defaults on a regular basis. Examples of such defaults include the failure of I.D. Herstatt in 1974 and the Bank of New York overnight shortfall of $22.6 billion in 1985. Systemwide meltdowns also occur. For example, consider the collapse of the Tokyo real estate market, the bankruptcy and public bailout of American S&Ls to the cost of about $500 billion, the Venezuelan bank crisis of 1994, and the Long Term Capital bailout associated with the Russia’s sovereign debt default. One of the most interesting failures of a tightly interconnected clearing system was the 1982 collapse of