

On dependence consistency of CoVaR and some other systemic risk measures

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Abstract

This paper is dedicated to the consistency of systemic risk measures with respect to stochastic dependence. It compares two alternative notions of Conditional Value-at-Risk (CoVaR) available in the current literature. These notions are both based on the conditional distribution of a random variable Y given a stress event for a random variable X , but they use different types of stress events. We derive representations of these alternative CoVaR notions in terms of copulas, study their general dependence consistency and compare their performance in several stochastic models. Our central finding is that conditioning on $X \geq \text{VaR}_\alpha(X)$ gives a much better response to dependence between X and Y than conditioning on $X = \text{VaR}_\alpha(X)$. We prove general results that relate the dependence consistency of CoVaR using conditioning on $X \geq \text{VaR}_\alpha(X)$ to well established results on concordance ordering of multivariate distributions or their copulas. These results also apply to some other systemic risk measures, such as the Marginal Expected Shortfall (MES) and the Systemic Impact Index (SII). We provide counterexamples showing that CoVaR based on the stress event $X = \text{VaR}_\alpha(X)$ is not dependence consistent. In particular, if (X, Y) is bivariate normal, then CoVaR based on $X = \text{VaR}_\alpha(X)$ is not an increasing function of the correlation parameter. Similar issues arise in the bivariate t model and in the model with t margins and a Gumbel copula. In all these cases, CoVaR based on $X \geq \text{VaR}_\alpha(X)$ is an increasing function of the dependence parameter.

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