Cascading Defaults and Systemic Risk of a Banking Network

Jin-Chuan DUAN & Changhao ZHANG
Risk Management Institute & NUS Business School
National University of Singapore
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**Key Contributions**

- A comprehensive structural model for a banking system to (1) differentiate between **systemic risk** and **systematic risk**, and (2) define **systemic exposure** and **systemic fragility**.

- **Marginal systemic risk** measures a bank’s individual contribution to systemic risk to identify systemically-important financial institutions (SIFIs).

- Compute systemic risk via an efficient **bridge-sampling** technique that only samples under rare future events.
Key Contributions (continued)

- Demonstrate the model with a network of 15 British banks, which reveals three insights:
  - Systemic risk was indeed present during the financial crisis, particularly in terms of systemic fragility.
  - Shocks to systematic risk factors (vs. banks' idiosyncratic elements) are more likely to drive cascading defaults and cause higher systemic risk.
  - Netting (due to the small magnitude of interbank exposures) does not have too big an impact on reducing systemic risk.
Outline

• Systemic risk vs. systematic risk
• Asset and liability dynamics
• Interbank credit links and cascading defaults
• Measures of systemic risk
• Dynamic scenario analysis and stress-testing
• An example of the UK banking network
• Determining SIFIs
Systemic and Systematic Risks

Systemic risk

• Schwarcz (2008): “an economic shock such as market or institutional failure (that) triggers the failure of a chain of markets or institutions, or a chain of significant losses to financial institutions...”
• Also, Caruana (2010), Bandt and Hartmann (2000).
• Arises due to interconnections.

Systematic risk

• Risk that cannot be diversified away and therefore affects most, if not all, market participants.
• Arises from exposures to common risk factors.
Systemic vs Systematic Risks

Systemic Risk: Arises due to interconnections
Systematic Risk: Arises from exposures to common risk factors
## Existing Literature

### Non-network models
- **Acharya, et al (2010), Brownlees and Engle (2012):** Extent of capital shortfall in the economy caused by the drop in the market
- **Hautsch, et al (2011), Adrian and Brunnermeier (2011):** VaR-based
- **Billio, et al (2011):** Granger causality in a PCA setup
- **Huang, et al (2010):** CDS premium for systemic distress

### Network models (without capital structure)
- **Giesecke and Weber (2005):** Interacting particle systems
- **Horst (2005):** Mean-field theory

### Network models (with capital structure)
- **Duan and Zhang (2013) (This paper)**

### Weaknesses of the non-network approaches
- Rely on correlations from past data, thus unable to separate systemic from systematic risk.
- Lack of capital structure in terms of the interplay between exogenous shocks and the bankruptcy trigger.
## Existing Literature

### Non-network models
- Billio, et al (2011): Granger causality in a PCA setup

### Network models (without capital structure)
- Giesecke and Weber (2005): Interacting particle systems
- Horst (2005): Mean-field theory

### Network models (with capital structure)
- Duan and Zhang (2013) (This paper)

### Weaknesses of the network (without capital structure) approaches
- Strong assumption of homogeneity; banks’ asset positions not characterised.
- Source of exogenous shocks and their interplay with the network is NOT explicit.
- Abstraction of financial systems into physical systems do not lend easily to interpretation.
## Existing Literature

<table>
<thead>
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</table>

### Strengths of our approach vis-à-vis Anand, *et al.* (2013)

- Accommodates heterogeneity in a banking network in a natural way via banks’ assets and liabilities with the dynamics that are structured into **systematic** and **idiosyncratic** risk components.
- Cascading defaults can be endogenously introduced through insolvency moderated with other considerations (i.e., not strict insolvency).
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</tr>
</tbody>
</table>

### Strengths of our approach vs Nier, *et al* (2007), and Marquez and Martinez (2009)

- Explicitly model the asset-liability dynamics of each bank with systematic and idiosyncratic components.
- Soft, rather than hard, insolvency trigger.
- Based on DTD measure for financial firms; shown to be consistently and significantly related to likelihood of default.
# Channels of Systemic Risk

## Key Channels*

- Direct bilateral exposures
- Correlated exposures to common risk
- Feedback effects from fire-sales
- Informational contagion

## Others

- Allen & Gale (2000): Forced liquidation due to excess liquidity demand
- Rosenthal (2011): Post-bankruptcy rehedging game

Asset-Liability Dynamics

\[
\frac{dV_{it}}{V_{it-}} = \mu_idt + \sum_{k=1}^{K} \beta_{ik}df_{kt} + \sigma_idW_{it} + Y^{(c)}_i dN^{(c)}_t + Y_idN_{it} + \frac{dL_{it}}{V_{it-}}
\]

\[
\frac{dL_{it}}{L_{it-}} = \varphi_idt + \sum_{k=1}^{K} \gamma_{ik}df_{kt} + \nu_idB_{it}.
\]

Common risk factors
- Interest rate term spread
- Stock market index
- FX
Asset-Liability Dynamics (continued)

\[\frac{dV_{it}}{V_{it-}} = \mu_idt + \sum_{k=1}^{K} \beta_{ik}df_{kt} + \sigma_idW_{it} + Y_i^{(c)}dN_t^{(c)} + Y_idN_{it} + \frac{dL_{it}}{V_{it-}}\]

\[\frac{dL_{it}}{L_{it-}} = \varphi_idt + \sum_{k=1}^{K} \gamma_{ik}df_{kt} + \nu_idB_{it}.\]

Common operational/credit events
Asset-Liability Dynamics (continued)

\[
\frac{dV_{it}}{V_{it-}} = \mu_i dt + \sum_{k=1}^{K} \beta_{ik} df_{kt} + \sigma_i dW_{it} + Y^{(c)}_i dN_t^{(c)} + Y_i dN_{it} + \frac{dL_{it}}{V_{it-}}
\]

\[
\frac{dL_{it}}{L_{it-}} = \varphi_i dt + \sum_{k=1}^{K} \gamma_{ik} df_{kt} + \nu_i dB_{it}.
\]

Asset-liability adjustment
Representation of Interbank Claims

Interbank exposure matrix

\[ \Pi = \begin{bmatrix} 0 & \pi_{12} & \pi_{13} \\ \pi_{21} & 0 & \pi_{23} \\ \pi_{31} & \pi_{32} & 0 \end{bmatrix} \]

→ Bank 2’s claim on Bank 3 (dollars)

Representation through Network Configuration Matrix

\[ Q = \begin{bmatrix} 0 & q_{12} & q_{13} \\ q_{21} & 0 & q_{23} \\ q_{31} & q_{32} & 0 \end{bmatrix} \]

→ Fraction of Bank 3’s liabilities as a specific obligation to Bank 2

\[ \Pi = Q'L_t \quad \text{where} \quad L_t \quad \text{is a diagonal bank liabilities matrix} \]
Bilateral Netting

- **Close-out netting** terminates obligations with the defaulting party and combines the replacement values of multiple transactions into a single net payable or receivable.

- **Default settlements**
  - Zero netting: Defaulter demands payments owed by other banks while it defaults on its own obligations.
  - Bilateral netting: Both parties will first net out their interbank positions.

- Mathematically:
  
  \[
  \omega_{jit} = \pi_{jit} - \psi \min(\pi_{ijt}, \pi_{jit})
  \]

  - Net exposure
  - Gross exposure
  - Netting
Bank Default Clearing Process

- Common market risk factors, bank assets and liabilities are evolved dynamically.
- Bank default is set off by a soft insolvency trigger to reflect other considerations such as liquidity.
  - A logistic function of the other-liabilities-incorporated distance-to-default measure (Duan and Wang, 2012) is used to determine default likelihood.
- Upon default:
  - Defaulting bank (DB) claims against non-defaulting banks (NDB); liabilities of NDB are marked down (and therefore assets as well). Bilateral netting kicks in.
  - DB’s marked-down assets are subject to a 10% fire sale discount.
Bank Default Clearing Process (continued)

In mathematical terms:

\[
D^{(l)}_{i\tau_k} = \begin{cases} 
1 & \text{if } D^{(l-1)}_{i\tau_k} = 1 \\
0 & \text{otherwise}
\end{cases}
\]

\[
\hat{V}^{(l)}_{i\tau_k} = \max \left[ 0, V^{(l)}_{i\tau_k} - \sum_{j \neq i}^M 1_{\{D^{(l-1)}_{j\tau_k} = 0 \land D^{(l)}_{j\tau_k} = 1\}} \psi \min (\pi_{ji\tau_k}, \pi_{ij\tau_k}) \right]
\]

\[
\hat{L}^{(l)}_{i\tau_k} = \hat{L}^{(l)}_{i\tau_k} - \sum_{j \neq i}^M 1_{\{D^{(l-1)}_{j\tau_k} = 0 \land D^{(l)}_{j\tau_k} = 1\}} \psi \min (\pi_{ji\tau_k}, \pi_{ij\tau_k})
\]

\[
V^{(l+1)}_{i\tau_k} = \begin{cases} 
\hat{V}^{(l)}_{i\tau_k} - \sum_{j \neq i}^M 1_{\{D^{(l-1)}_{j\tau_k} = 0 \land D^{(l)}_{j\tau_k} = 1\}} \left[ \pi_{ji\tau_k} + \omega_{ij\tau_k} \left( 1 - \frac{\phi_j \hat{V}^{(l)}_{j\tau_k}}{\hat{L}^{(l)}_{j\tau_k}} \right) \right] & \text{if } D^{(l)}_{i\tau_k} = 0 \\
\hat{V}^{(l)}_{i\tau_k} & \text{otherwise}
\end{cases}
\]

\[
V^{(l+1)}_{i\tau_k} = \max \left( 0, V^{(l+1)}_{i\tau_k} \right)
\]

\[
L^{(l+1)}_{i\tau_k} = \begin{cases} 
\hat{L}^{(l)}_{i\tau_k} - \sum_{j \neq i}^M 1_{\{D^{(l-1)}_{j\tau_k} = 0 \land D^{(l)}_{j\tau_k} = 1\}} \pi_{ji\tau_k} & \text{if } D^{(l)}_{i\tau_k} = 0 \\
\hat{L}^{(l)}_{i\tau_k} & \text{otherwise}
\end{cases}
\]
Example of Cascading Defaults (No Netting)

- Banking Configuration matrix

\[
Q = \begin{bmatrix}
0 & 0.15 & 0.15 & 0.10 & 0.10 \\
0.08 & 0 & 0.12 & 0.20 & 0.10 \\
0.18 & 0.02 & 0 & 0.15 & 0.15 \\
0.05 & 0.15 & 0.12 & 0 & 0.18 \\
0.20 & 0.10 & 0.20 & 0 & 0 \\
\end{bmatrix}
\]

- Liabilities

\[
L_t = \begin{bmatrix}
200 & 0 & 0 & 0 & 0 \\
0 & 200 & 0 & 0 & 0 \\
0 & 0 & 100 & 0 & 0 \\
0 & 0 & 0 & 300 & 0 \\
0 & 0 & 0 & 0 & 100 \\
\end{bmatrix}
\]
Example of Cascading Defaults (No Netting) (continued)

- **Interbank exposures**
  
  \[ \Pi_t = \begin{bmatrix}
  0 & 16 & 18 & 15 & 20 \\
  30 & 0 & 2 & 45 & 10 \\
  30 & 24 & 0 & 36 & 20 \\
  20 & 40 & 15 & 0 & 0 \\
  20 & 20 & 15 & 54 & 0
\end{bmatrix} \]

- **Initial state**
  
  
<table>
<thead>
<tr>
<th>Bank 1</th>
<th>Bank 2</th>
<th>Bank 3</th>
<th>Bank 4</th>
<th>Bank 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset value:</td>
<td>225</td>
<td>195</td>
<td>120</td>
<td>305</td>
</tr>
<tr>
<td>Liability value:</td>
<td>200</td>
<td>200</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>Default indicator:</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

  Bank 2 in default

- **Fire sale discount factor = 0.8 and assume a hard insolvency default trigger.**
Example of Cascading Defaults (No Netting)
(continued)

First iteration

<table>
<thead>
<tr>
<th>Bank</th>
<th>Bank 2</th>
<th>Bank 3</th>
<th>Bank 4</th>
<th>Bank 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset value:</td>
<td>191.48</td>
<td>195</td>
<td>112.72</td>
<td>251.2</td>
</tr>
<tr>
<td>Liability value:</td>
<td>170</td>
<td>200</td>
<td>98</td>
<td>255</td>
</tr>
<tr>
<td>Default indicator:</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Down by 30 due to Bank 2’s claim on Bank 1

Down by 33.52 because:
(1) offset of 30;
(2) unrecovered claim of 3.52 on Bank 2

\[
\pi_{21} + \pi_{12} \left(1 - \frac{\phi_{22}V_2}{L_2}\right) = 30 + 16 \times \left(1 - \frac{0.8 \times 195}{200}\right) = 33.52
\]
Example of Cascading Defaults (No Netting) (continued)

Similar computations for other banks (3, 4 & 5)

<table>
<thead>
<tr>
<th></th>
<th>Bank 1</th>
<th>Bank 2</th>
<th>Bank 3</th>
<th>Bank 4</th>
<th>Bank 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset</td>
<td>191.48</td>
<td>195</td>
<td>112.72</td>
<td>251.2</td>
<td>100.6</td>
</tr>
<tr>
<td>Liability</td>
<td>170</td>
<td>200</td>
<td>98</td>
<td>255</td>
<td>90</td>
</tr>
<tr>
<td>Default indicator</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Cascading defaults begin
Example of Cascading Defaults (No Netting)
(continued)

Second iteration

Bank 4 triggers Bank 5’s default

<table>
<thead>
<tr>
<th></th>
<th>Bank 1</th>
<th>Bank 2</th>
<th>Bank 3</th>
<th>Bank 4</th>
<th>Bank 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset</td>
<td>168.30</td>
<td>195</td>
<td>90.09</td>
<td>251.2</td>
<td>89.16</td>
</tr>
<tr>
<td>Liability</td>
<td>150</td>
<td>200</td>
<td>83</td>
<td>255</td>
<td>90</td>
</tr>
<tr>
<td>Default</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Third iteration

Revision to asset and liabilities of other banks (1 and 3) do not trigger further defaults

<table>
<thead>
<tr>
<th></th>
<th>Bank 1</th>
<th>Bank 2</th>
<th>Bank 3</th>
<th>Bank 4</th>
<th>Bank 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset</td>
<td>144.15</td>
<td>195</td>
<td>70.94</td>
<td>251.2</td>
<td>89.16</td>
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<tr>
<td>Liability</td>
<td>130</td>
<td>200</td>
<td>68</td>
<td>255</td>
<td>90</td>
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<tr>
<td>Default</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Example of Cascading Defaults (Full Netting)

- Banking Configuration matrix (as before)

\[
Q = \begin{bmatrix}
0 & 0.15 & 0.15 & 0.10 & 0.10 \\
0.08 & 0 & 0.12 & 0.20 & 0.10 \\
0.18 & 0.02 & 0 & 0.15 & 0.15 \\
0.05 & 0.15 & 0.12 & 0 & 0.18 \\
0.20 & 0.10 & 0.20 & 0 & 0
\end{bmatrix}
\]

- Liabilities (as before)

\[
L_t = \begin{bmatrix}
200 & 0 & 0 & 0 & 0 & 0 \\
0 & 200 & 0 & 0 & 0 & 0 \\
0 & 0 & 100 & 0 & 0 & 0 \\
0 & 0 & 0 & 300 & 0 & 0 \\
0 & 0 & 0 & 0 & 100 & 0
\end{bmatrix}
\]
Example of Cascading Defaults (Full Netting)
(continued)

- Interbank exposures (as before)
  \[
  \Pi_t = \begin{bmatrix}
  0 & 16 & 18 & 15 & 20 \\
  30 & 0 & 2 & 45 & 10 \\
  30 & 24 & 0 & 36 & 20 \\
  20 & 40 & 15 & 0 & 0 \\
  20 & 20 & 15 & 54 & 0 \\
  \end{bmatrix}
  \]

- Initial state (as before)

- Bank 2 in default

- Fire sale discount factor = 0.8 and assume a hard insolvency default trigger.
Example of Cascading Defaults (Full Netting) (continued)

- **Net exposures**

\[
\Omega = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
14 & 0 & 0 & 5 & 0 \\
12 & 22 & 0 & 21 & 5 \\
5 & 0 & 0 & 0 & 0 \\
0 & 10 & 0 & 54 & 0 \\
\end{bmatrix}
\]

- **No further defaults after the first iteration**

<table>
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<tr>
<td></td>
<td>195</td>
<td>127</td>
<td>112.93</td>
<td>260</td>
<td>102.7</td>
</tr>
<tr>
<td>Liability value:</td>
<td>170</td>
<td>132</td>
<td>98</td>
<td>255</td>
<td>90</td>
</tr>
<tr>
<td>Default indicator:</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
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Down by 30 due to Bank 2’s claim on Bank 1

Down by 30 because:
1. offset of 30
2. no unrecovered claim on Bank 2 because of full netting
**Systemic Risk Measures**

- **Total Risk**
  - Systemic Risk
  - Systematic Risk
  - T-Idiosyncratic Risk

**Exposure**: Expected uncovered losses

**Fragility**: Expected fraction of bank defaults

Incremental risk due to banking network $Q$, against a null benchmark 0 with no interbank linkages.

Incremental risk of the null benchmark 0 against a system driven purely by idiosyncratic shocks.

Total asset risk converted into pure idiosyncratic risk. System’s total exposure and fragility when there are no interbank linkages and bank assets are totally uncorrelated.
**Systemic Risk Measures** (continued)

- **Systemic exposure**: Incremental uncovered loss due to the banking network $Q$ against a null benchmark network $0$ that has no interbank linkages.

  $$TL^{(Q)}_{[0,T]} = \sum_{i=1}^{M} \int_{0}^{T} e^{-\int_{0}^{t} r_s ds} \left( L_{it}^{(Q)} - \phi_i V_{it}^{(Q)} \right) dD_{it}^{(Q)}$$

  $$SystemicExp_{[0,T]}^{(Q)}(A) = E_0 \left[ TL^{(Q)}_{[0,T]} - TL^{(0)}_{[0,T]} \right] | A$$

- **Systematic exposure**: Incremental uncovered loss of the null benchmark network $0$ against a system driven purely by idiosyncratic shocks.

  $$SystemicExp_{[0,T]}^{(0)}(A) = E_0 \left[ TL^{(0)}_{[0,T]} - TL^{(0,*)}_{[0,T]} \right] | A$$
Systemic Risk Measures (continued)

- **Systemic Fragility**: Expected proportion of bank defaults due to banking network \( Q \), benchmarked against \( 0 \).

\[
SystemicFra_{[0,T]}^{(Q)}(A) = E_0 \left[ \frac{\sum_{i=1}^{M} \int_{0}^{T} dD_{is}^{(Q)} - \sum_{i=1}^{M} \int_{0}^{T} dD_{is}^{(0)}}{M} \right] A
\]

- **Systematic Fragility**: Expected proportion of bank defaults of the null benchmark network \( 0 \) against a system driven purely by idiosyncratic shocks.

\[
SystematicFra_{[0,T]}^{(0)}(A) = E_0 \left[ \frac{\sum_{i=1}^{M} \int_{0}^{T} dD_{is}^{(0)} - \sum_{i=1}^{M} \int_{0}^{T} dD_{is}^{(0,*)}}{M} \right] A
\]
Totally-Idiosyncratic Risk

• Total asset risk converted into entirely idiosyncratic risk
  – System’s total exposure and fragility when there are no interbank linkages and bank assets are totally uncorrelated.

\[
\text{T-IdiosyncraticExp}_{[0,T]}^{(0)}(A) = E_0 \left[ TL_{[0,T]}^{(0,*)} | A \right]
\]

\[
\text{T-IdiosyncraticFra}_{[0,T]}^{(0)}(A) = E_0 \left[ \frac{\sum_{i=1}^{M} \int_0^T dD_{i,s}^{(0,*)}}{M} | A \right]
\]
Determining SIFIs

• “Too-big-to-fail” and “Too-connected-to-fail”
  – FSB: “Financial institutions whose distress or disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity.”

• Our model is particularly suited for this purpose because
  – Systemic interconnectedness is captured by the banking configuration matrix (Q)
  – Size, complexity and loss absorption capacity are captured by asset-liability dynamics
Determining SIFIs (continued)

- **Marginal systemic risk measures**: the increase in systemic risk attributable to a particular bank.

\[
\text{MargSystemicFra}_{[0,T]}^{(Q,i)}(A) = E_0 \left[ \sum_{i=1}^{M} \int_0^T dD_{is}^{(Q)} - \sum_{i=1}^{M} \int_0^T dD_{is}^{(Q-i)} \right] \left| A \right|
\]

\[
\text{MargSystemicExp}_{[0,T]}^{(Q,i)}(A) = E_0 \left[ TL_{[0,T]}^{(Q)} - TL_{[0,T]}^{(Q-i)} \right] \left| A \right|
\]

**Note**: \( Q_{-i} \) stands for the banking configuration matrix with the \( i^{th} \) row and column set to zeros.
Dynamic Scenario Analysis

• Conditioning event A
  – For example, stock market index drops by at least 40% over a period of 6 months.

• Key features sought:
  – Price fall is not static but takes place over time in a random fashion in accordance with the prescribed dynamics.
  – Other common risk factors (say, term spread and exchange rate) move in a consistent and correlated manner.
  – Wide variety of scenarios are allowed to play out; may capture scenarios under which a system is particularly vulnerable.

• Implemented through Gaussian bridge sampling.
Systemic Risk of the UK Banking Network

• Three dynamic common risk factors:
  – UK FTSE 100 Index
  – Trade-weighted GBP
  – GBP LIBOR Spread (12m-1m)

• One latent factor is included through a PCA of the residuals to capture remaining systematic effects of the banks’ asset value processes.
  – Without latent factor: $R^2$ ranged from 6% to 30%.
  – With latent factor: $R^2$ ranged from 24% to 70%.

• 500,000 sets of simulated paths
Systemic Risk of the UK Banking Network (continued)

Conditioning event
- The stock index falls at least 40% over a 6 month period.
- Historically, a 40% drop occurred over the 2008-2009 period.

Banking Network Configuration
- 15 British banks, including all major ones.
- Network configuration setup from cross-sectional distribution of interbank exposures.

<table>
<thead>
<tr>
<th>Table 1: Summary statistics of interbank claims</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>To an individual bank</td>
</tr>
<tr>
<td>Total claims of a bank</td>
</tr>
</tbody>
</table>
Systemic Risk of the UK Banking Network (continued)

Asset dynamics

- Market value of assets obtained from the Credit Research Initiative (CRI) of Risk Management Institute (RMI), National U of Singapore, which are obtained via an MLE method (including other liabilities) suitable for financial firms.
- Assets respond positively to the FTSE100 index and GBP. Response to interest rate term spread depends on the period in question.

Liability dynamics

- Adjusted liabilities obtained from RMI-CRI.
- Fixed over 6 months in simulation but changed quarterly with the availability of new financial statements.
Time Profile of Systemic Risk
Time Profile of Systemic Risk (continued)
Stock Market Shocks Drive Systematic Risk

- T-Idiosyncratic Fragility (A)
- Systematic Fragility (B)
- Systemic Fragility (C)
- Total (A+B+C)

2009-Q4

Unconditional 0 -0.1 -0.2 -0.3 -0.4

bps

RMI
Advancing Risk Management for Singapore and Beyond

Duan & Zhang (June 2015)
Cascading Defaults and Systemic Risk of a Banking Network
Stock Market Shocks Drive Systematic Risk

(continued)

- Idiosyncratic Exposure (A)
- Systematic Exposure (B)
- Systemic Exposure (C)
- Total (A+B+C)

2009-Q4
Systematic Risk as a Driver for Systemic Risk

![Graph showing the relationship between proportion of systematic risk and systemic fragility.](image-url)

- **Systematic + T-Idiosyncratic (A)**
- **Systemic Fragility (B)**
- **Total (A+B)**

Proportion of Systematic Risk

- **2009-Q4**
Systematic Risk as a Driver for Systemic Risk

2009-Q4
Determining SIFIs – an example

*Assumes average interbank exposures for all banks, for demonstration purpose only

![Chart showing marginal systemic fragility for various banks in 2009-Q4](chart.png)
Determining SIFIs – an example (continued)

*Assumes average interbank exposures for all banks, for demonstration purpose only.

2009-Q4
Extras
FTSE 100 Index Drift and Volatility
TWI GBP Drift and Volatility

Drift 
Volatility

Term Spread Volatility

*No mean reversion of spread observed in full sample regression*
Bridge Sampling Example (term spread)

\[
R_{t+i\Delta t} = a_{k,i}(R_t) + b_{k,i} R_{t+k\Delta t} + \epsilon_{k,i}
\]

\[
b_{k,i} = \frac{(1 - \kappa \Delta t)^{k-i} [1 - (1 - \kappa \Delta t)^{2i}]}{1 - (1 - \kappa \Delta t)^{2k}}
\]

\[
a_{k,i}(R_t) = \tilde{R} [1 - (1 - \kappa \Delta t)^i] + (1 - \kappa \Delta t)^i R_t - b_{k,i} \{ \tilde{R} [1 - (1 - \kappa \Delta t)^{k}] + (1 - \kappa \Delta t)^{k} R_t \}.
\]

\[
Var(\epsilon_{k,i}) = \frac{\eta^2 \Delta t [1 - (1 - \kappa \Delta t)^{2i}] [1 - (1 - \kappa \Delta t)^{2(k-i)}]}{[1 - (1 - \kappa \Delta t)^{2k}] [1 - (1 - \kappa \Delta t)^{2k}]}.
\]

\[
d \ln I_t = \delta_I dt + \eta_I dW_{It}
\]

\[
= \delta_I dt + \eta_I \rho_{I,R} dW_{Rt} + \eta_I \sqrt{1 - \rho_{I,R}^2} dZ_{It}
\]

\[
d \ln e_t = \delta_e dt + \eta_e dW_{et}
\]

\[
= \delta_e dt + \eta_e \rho_{e,R} dW_{Rt} + \eta_e \sqrt{1 - \rho_{e,R}^2} dZ_{et}
\]
Effects of Bilateral Netting

- Systematic + T-Idiosyncratic (A)
- Systemic Fragility (B)
- Total (A+B)

Bar chart showing the impact of bilateral netting on systemic fragility, with a scale for bilateral netting ranging from 0 to 1.
Effects of Bilateral Netting (continued)

- Systematic + T-Idiosyncratic (A)
- Systemic Exposure (B)
- Total (A+B)

Bar chart showing the effects of bilateral netting on GDP billions.
Effects of Bilateral Netting (continued)